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IIR Filters Using Stochastic Arithmetic

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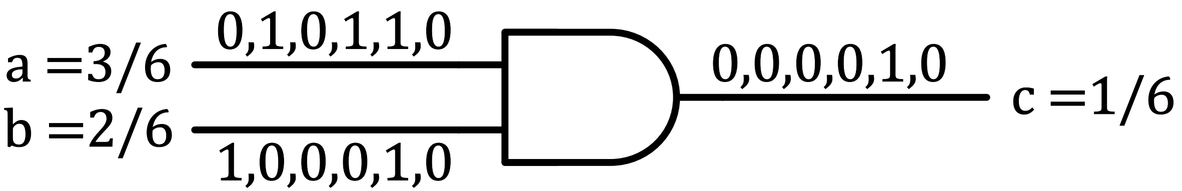
Outline

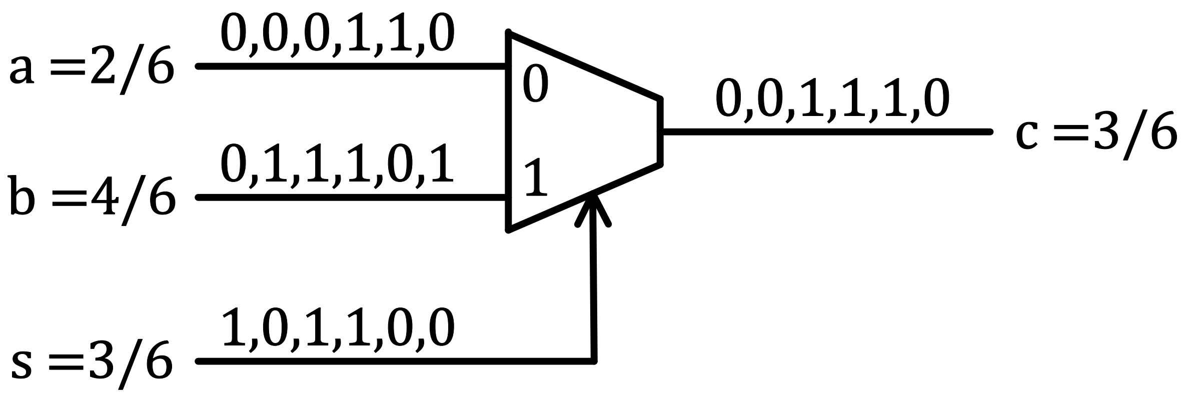
- Introduction
- Stochastic Filtering
- Experiments and Results
- Conclusions and Future Research

Introduction: Stochastic computing

- **Computing using random bit streams.**



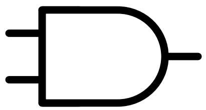
$$\boldsymbol{c = ab}$$

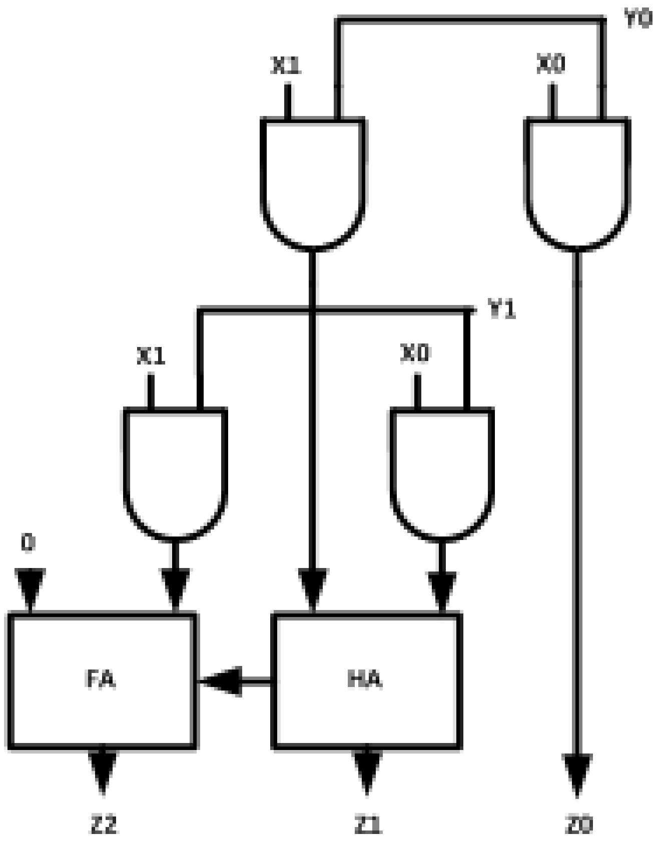


$$\boldsymbol{c = (1 - s)a + sb}$$

Stochastic computing: Area

Stochastic	Conventional
1 AND	7 AND + 3XOR + 1 OR





Stochastic computing: Fault Tolerance

- **Largest error due to a single bit flip in an M-bit binary fraction is $1/2$**

$$0.001 \longrightarrow 0.101$$

(0.125) (0.625)

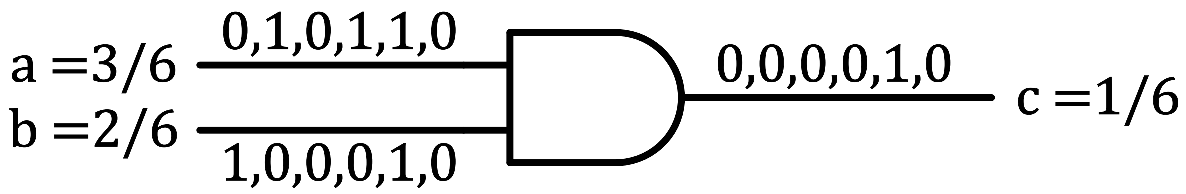
- **Largest error in a random bit stream of length 2^M due to a single bit flip is always 2^{-M}.**

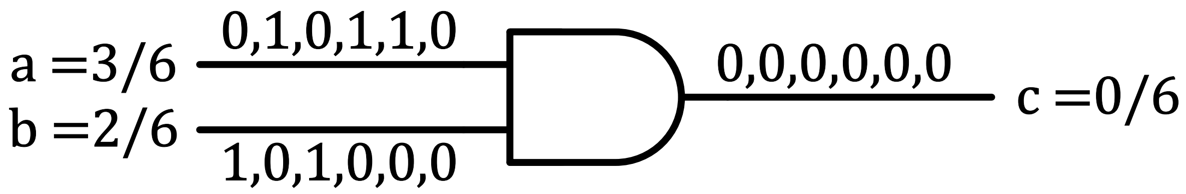
$$00100000 \longrightarrow 00100010$$

(0.125) (0.250)

Stochastic computing: Accuracy

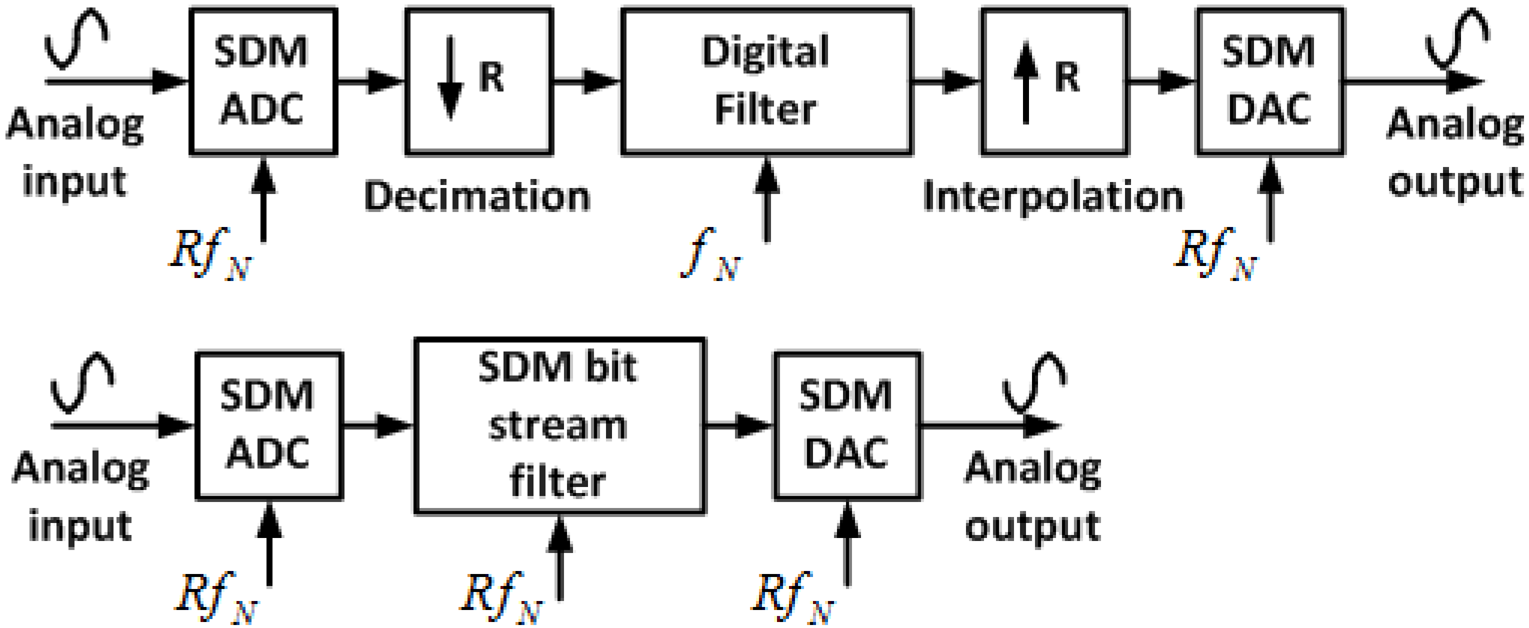
- **Random representation**



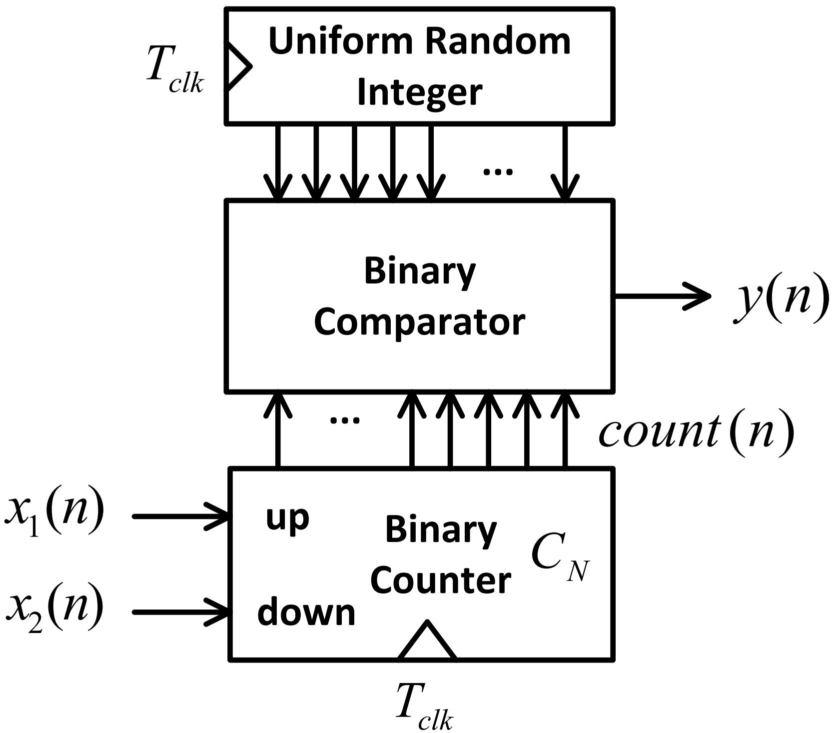


Stochastic Filtering: Motivation

- **Reduce Area**
  - **Multi-bit v/s bit stream**
  - **Interface filters**



Stochastic Integrator



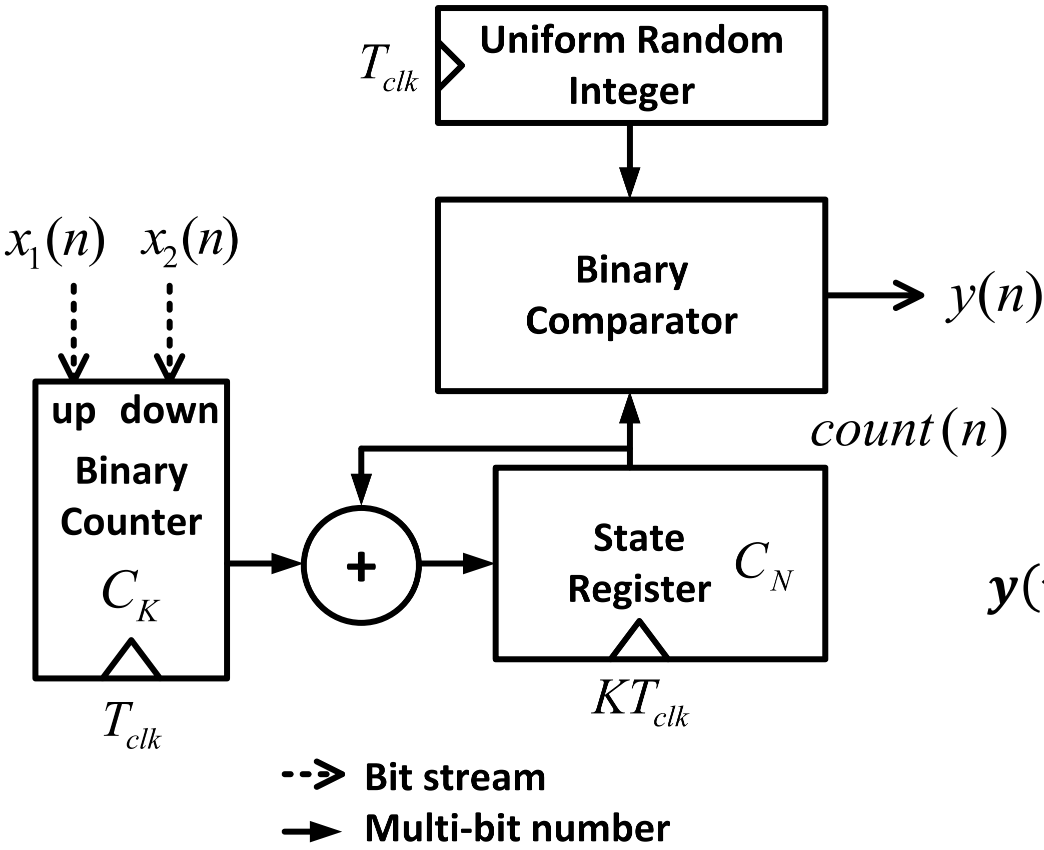
[Gaines 1969]

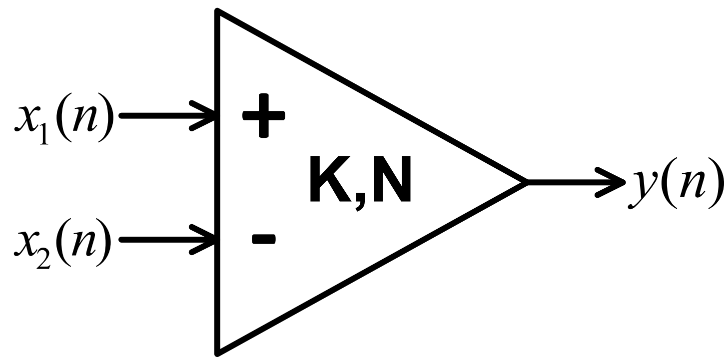
$$y(n+1) = y(n) + \frac{x_1(n) - x_2(n)}{N}$$

$$\left.\frac{Y(z)}{X_1(z)}\right|_{X_2=0} = \frac{1}{N(z-1)}$$

$$\left.\frac{Y(z)}{X_2(z)}\right|_{X_1=0} = \frac{-1}{N(z-1)}$$

Stochastic Integrator: Our Approach



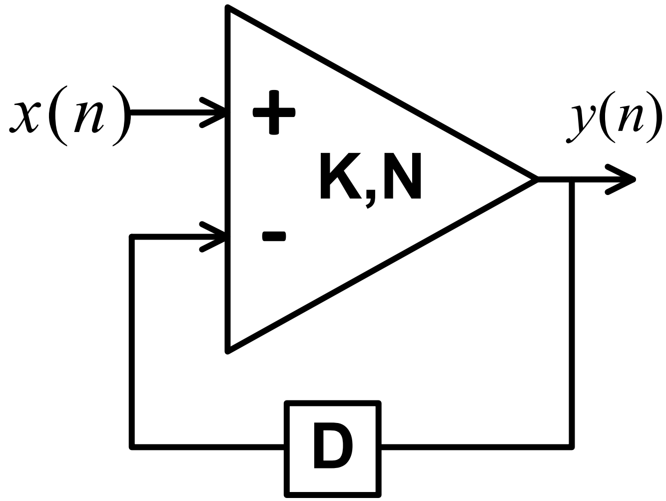


$$y(n+1) = y(n) + \frac{K(x_1(n) - x_2(n))}{N}$$

$$\left.\frac{Y(z)}{X_1(z)}\right|_{X_2=0} = \frac{K}{N(z-1)}$$

$$\left.\frac{Y(z)}{X_2(z)}\right|_{X_1=0} = \frac{-K}{N(z-1)}$$

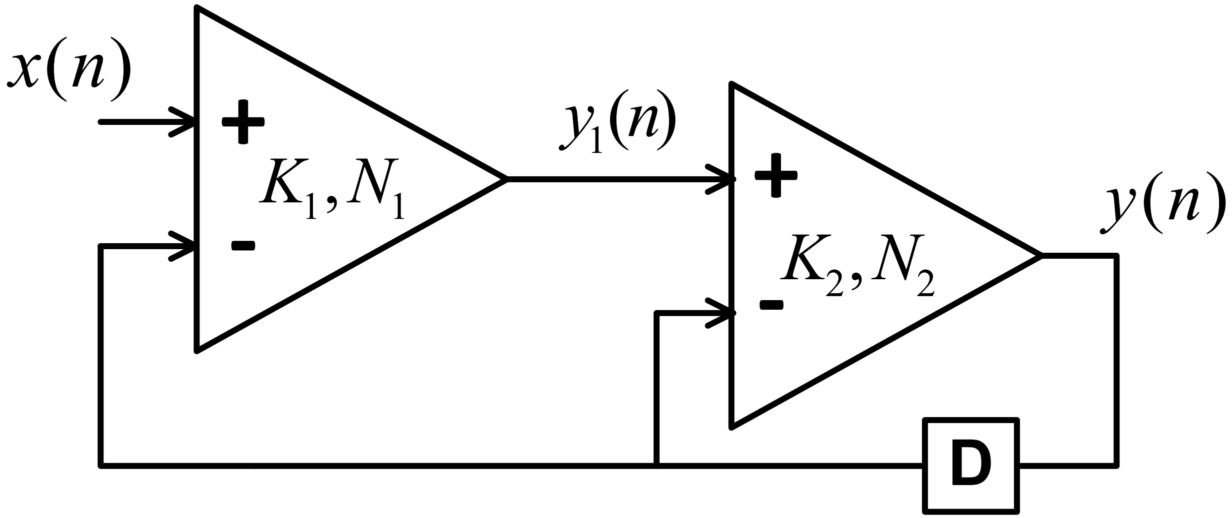
Order 1 Stochastic IIR Filter



$$H(z) = \frac{Y(z)}{X(z)} = \frac{K}{Nz + K - N}$$

$$p = 1 - K/N$$

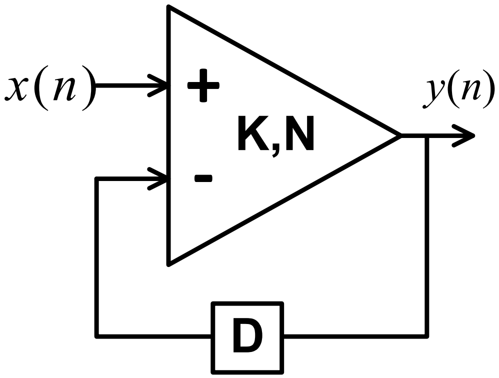
Order 2 Stochastic IIR Filter

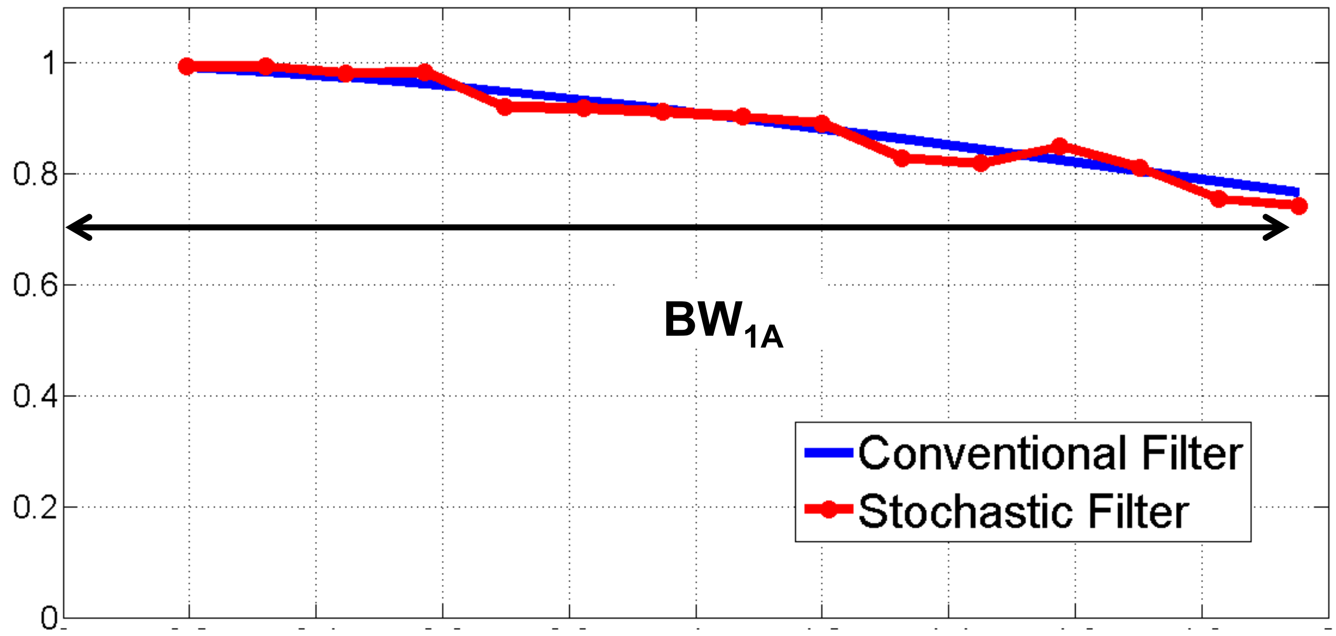


$$H(z) = \frac{K_1K_2}{N_1N_2z^2 + (N_1K_2 - 2N_1N_2)z + (N_1N_2 - N_1K_2 + K_1K_2)}$$

$$p_1, p_2 = 1 - \frac{K_2}{2N_2} \pm \sqrt{\frac{N_1^2K_2^2 - 4N_1N_2K_1K_2}{4N_1^2N_2^2}}$$

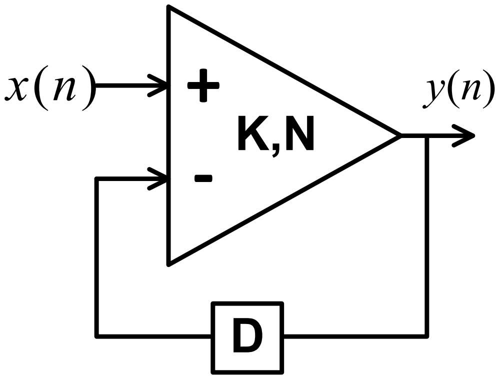
Experiments: Order 1 Stochastic IIR Filters

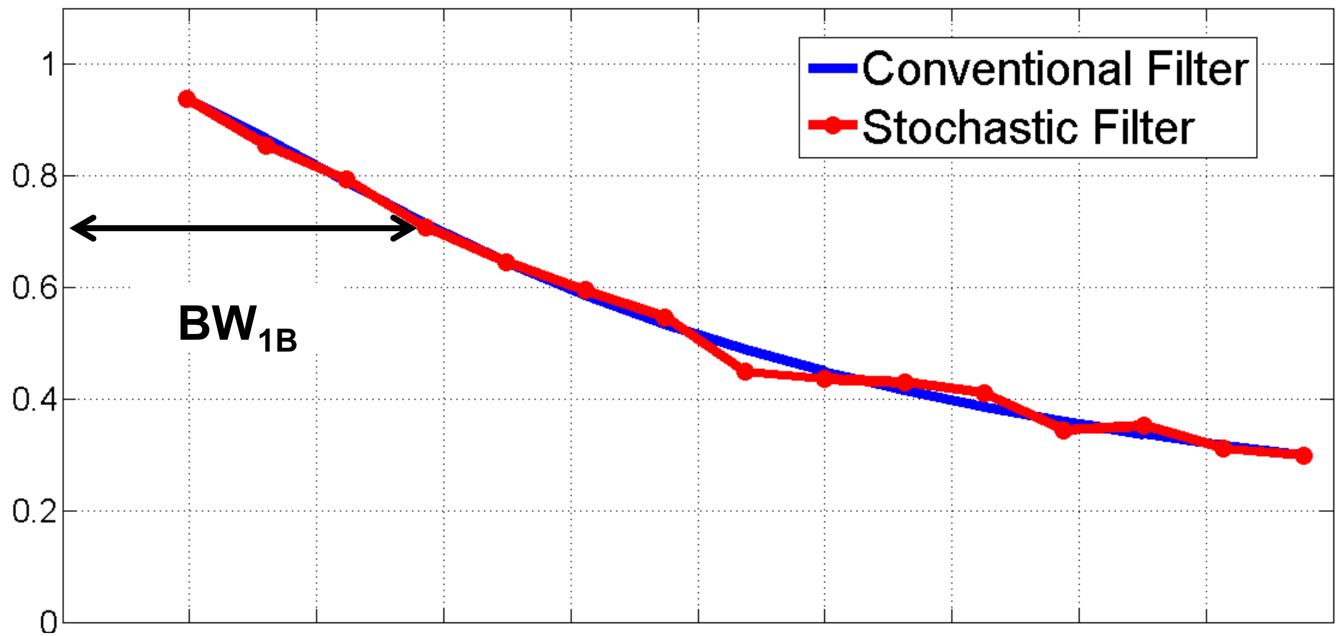




$$K_1 = 1, N_1 = 64, p_1 = 0.984$$

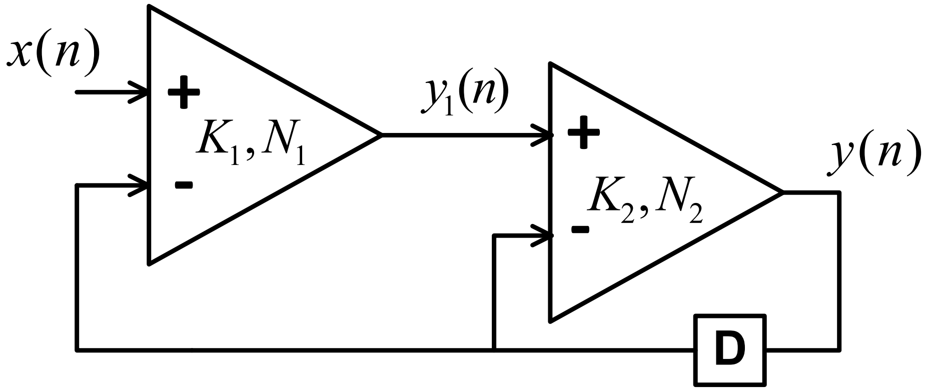
Experiments: Order 1 Stochastic IIR Filters



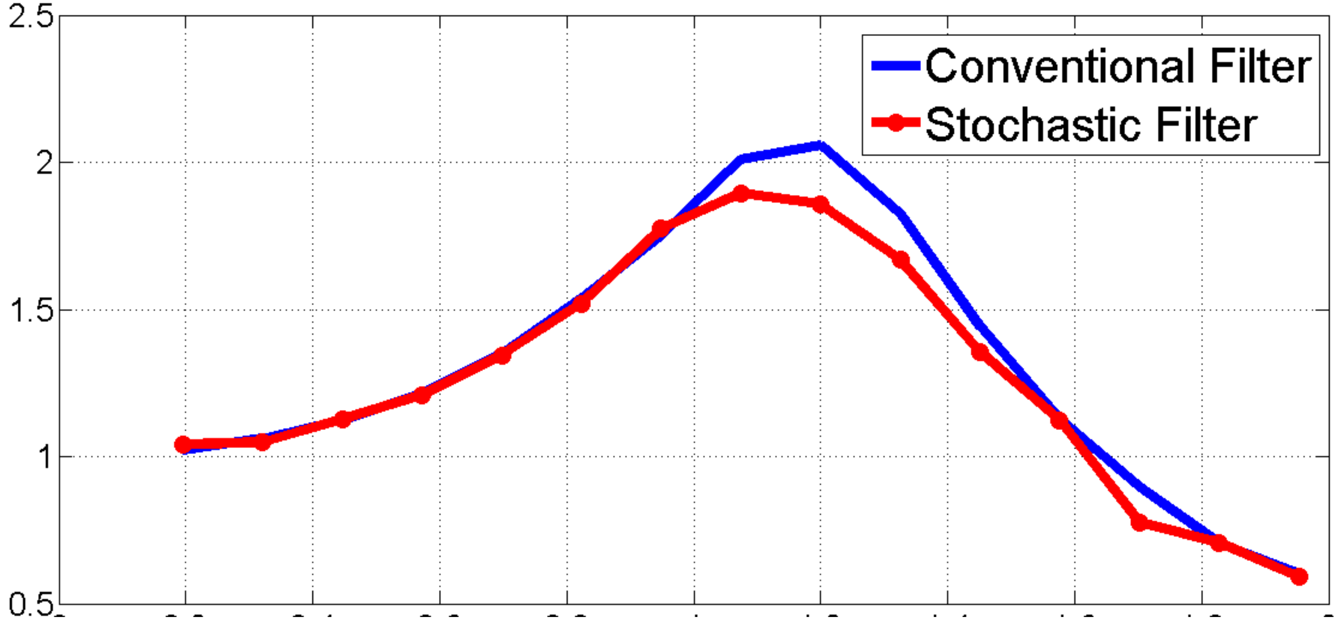


$$K_1 = 1, N_1 = 256, p_1 = 0.996$$

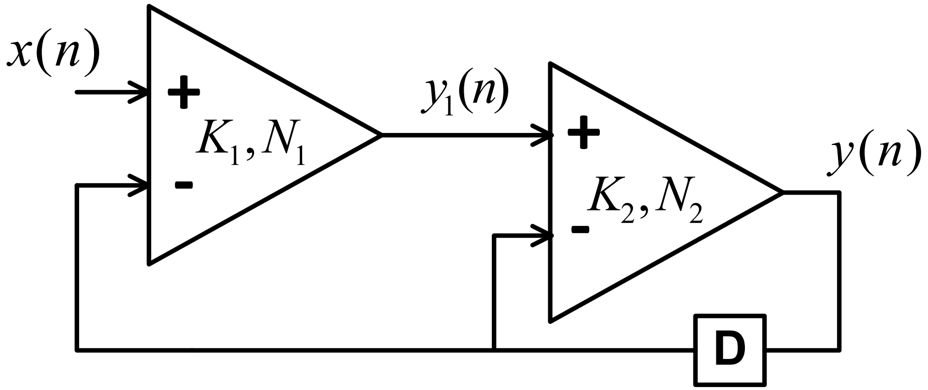
Experiments: Order 2 Stochastic IIR Filters



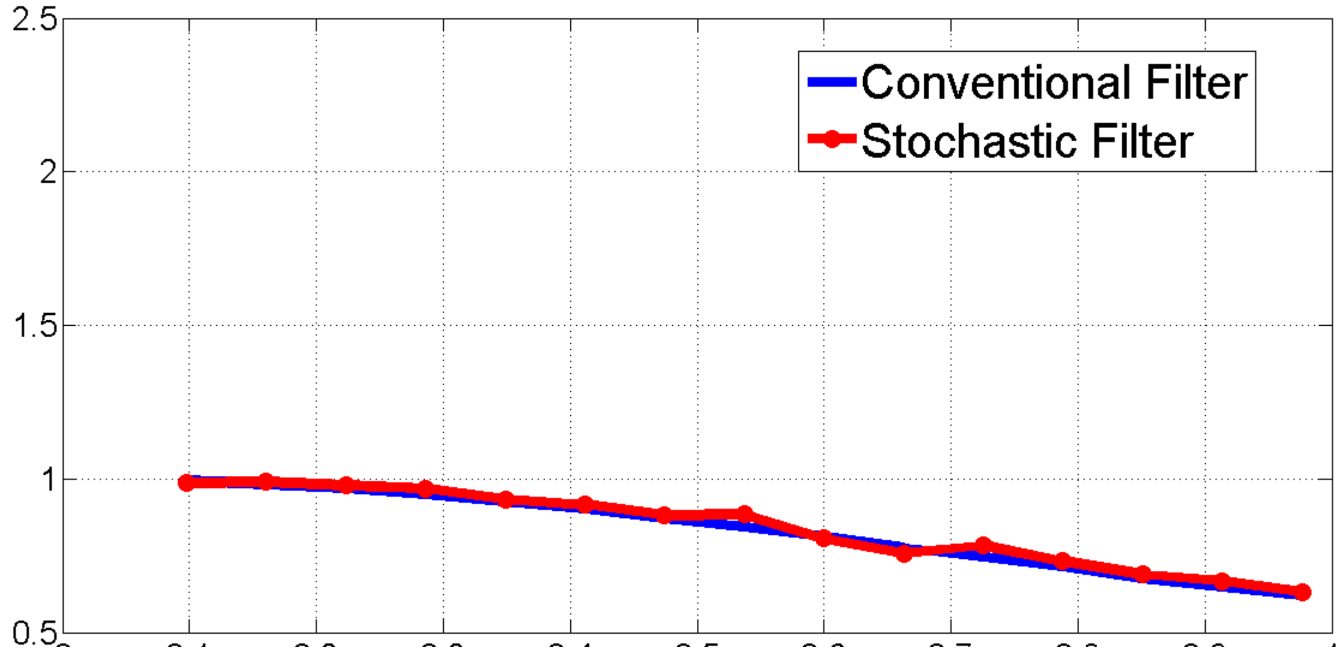
$$K_1 = 1, N_1 = 64,$$
$$K_2 = 1, N_2 = 256,$$
$$p_1 = 0.9980 + 0.0076i$$
$$p_2 = 0.9980 - 0.0076i$$



Experiments: Order 2 Stochastic IIR Filters



$$K_1 = 1, N_1 = 64,$$
$$K_2 = 1, N_2 = 256,$$
$$p_1 = 0.9922$$
$$p_2 = 0.9922$$



Hardware Results

Order	K_1	N_1	K_2	N_2	Conventional	Stochastic
1	1	64	-	-	554	118.4
1	1	256	-	-	897.2	156.1
2	1	64	1	256	1067.4	270.0
2	1	256	1	64	1248.3	270.0

Conclusions and Future Research

- **Stochastic filtering: area efficiency**
- **Complicated noise analysis**
- **Bandpass/Highpass filters**