Quiz # 2
Oct. 31 (Boo!), 2:35 - 3:40pm, in class

Books are allowed. Notes are allowed. Calculators are allowed – but only for arithmetic. Laptops, tablets, smart phones, etc., are not allowed.

1. Number Systems

(a) Suppose that the following bits represent positive numbers. Convert them to their decimal equivalents.
   i. $11111111_2$.
   ii. $01111111_2$.
   iii. $10101010_2$.
   iv. $10000001_2$.

(b) Suppose that the following bits represent positive and negative numbers in two's complement notation. Perform the operation indicated. If the result cannot be represented with 8 bits, say so. Otherwise give the result, both in binary and in decimal.
   i. $11111111_2 + 01111111_2$.
   ii. $01111111_2 + 0111110_2$
   iii. $11111110_2 - 11111111_2$.
   iv. $01111111_2 - 11001110_2$.

(c) Suppose that the following are all positive numbers. Perform the indicated conversion.
   i. $101011101101101000111101110_2$ to hexademical.
   ii. $4562071_8$ to hexademical.
   iii. $BADA55_{16}$ to octal.
2. Bit-Level Arithmetic

Consider the following C program.

```c
#include <stdio.h>
#include <stdlib.h>

int main(int argc, char **argv) {
    unsigned int v = atoi(argv[1]);
    unsigned int c;

    for (int i = 7; i >= 0; i --) {
        printf("%d", (v >> i) & 1);
    }
    printf("\n");

    for (c = 0; v > 0; c++) {
        v &= v - 1;
    }

    printf("%d\n", c);
}
```

What will it print out if it is called with the argument 12? With the argument 47? With the argument 111?

What does the program compute? Specifically, what is the relationship between the integer that it prints on the second line and the string of bits that it prints on the first line?
3. **Lowest Cost Paths Through Acyclic Graphs**

Find the minimum cost path from A to be B, where the cost is the sum of the numbers along that path.
4. **Knapsack Problem**

This problem will test your understanding of dynamic programming, an algorithmic archetype that we discussed in class.

Suppose that a ninja has broken into the Imperial Palace in Tokyo to steal valuable trinkets. He will catch the direct return flight with Delta to Minneapolis, but he’s very annoyed at Delta’s new fees for checked-in luggage. Out of principle, he refuses to pay such fees; everything that he steals from the Imperial Palace must fit in his backpack as carry-on. The tensile strength of his UMN backpack is such that he can safely pack 10 lbs.

The ninja assesses the value and weight of the trinkets to be as follows:

<table>
<thead>
<tr>
<th>item</th>
<th>value</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>￥1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>￥2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>￥4</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>￥1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>￥4</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>￥8</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>￥1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>￥8</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>￥16</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>￥32</td>
<td>8</td>
</tr>
</tbody>
</table>

What is the value of the best choice of trinkets?
5. **Sorting**

This problem tests your understanding of sorting (and, of course, recursion). Here is the code to a very common algorithm, quicksort(). What does the code print out?

```c
#include <stdio.h>

/* swap: interchange v[i] and v[j] */
void swap(int v[], int i, int j)
{
    int temp;
    temp = v[i];
    v[i] = v[j];
    v[j] = temp;
}

/* qsort: sort v[left]...v[right] into increasing order */
void qsort(int v[], int left, int right)
{
    int i, last;
    printf("left %d, right %d\n", left, right);

    /* do nothing if array contains fewer than two elements */
    if (left >= right)
        return;

    /* move "pivot" element to v[0] */
    swap(v, left, (left + right)/2);
    last = left;

    /* partition */
    for (i = left + 1; i <= right; i++)
        if (v[i] < v[left])
            swap(v, ++last, i);

    /* restore "pivot" element */
    swap(v, left, last);
    qsort(v, left, last-1);
    qsort(v, last+1, right);
}
```

for (i = 0; i <= 9; i++)
    printf("%d ", v[i]);
    printf("\n");

int main(int argc, char **argv) {

    int i;
    int v[10] = {1, 3, 2, 4, 6, 5, 8, 7, 9, 0};
    qsort(v, 0, 9);

    for (i = 0; i <= 9; i++)
        printf("%d ", v[i]);
    printf("\n");

}